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TEMPERATURE DEPENDENCE OF THE AMPLITUDE RATIO, ASSOCIATED WITH THE ZERO FREQUENCY CRITICAL BULK VISCOSITY

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ABSTRACT

It was shown by Onuki that the zero frequency bulk viscosity is associated with a universal amplitude ratio that was calculated to be around 0.10. We study the bulk viscosity from the physical stand point of view of sound attenuation and show that the amplitude ratio, associated with the critical bulk viscosity, is not truly universal. It has a logarithmic correction instead.

INTRODUCTION

In 1997, it was shown by Onuki [1,2] that the zero frequency bulk viscosity η_B diverges near the critical point of a liquid-gas system and it was established that if the critical point fluctuations decay with the relaxation rate $\Gamma=\Gamma_0\xi^{-(3+x\_\eta)}$, where ξ is the correlation length, x_η is the very small viscosity exponent, and C_s is the zero frequency sound velocity, then there is a universal combination, given by

$$\frac{\eta_B}{\rho}\frac{\Gamma_0}{C_S^2\xi^{(3+x_\eta)}}=R_B \tag{1}$$

From an ε - expansion, Onuki found that R_B is to be 0.087 and from a three dimensional caculation he found it to be 0.13. In this paper, we will show that R_B is not a constant. Instead, there is a temperature correction to this universal combination in detailed calculations.

Since the bulk viscosity is a slightly nonstandard concept, we will explain our point of view in a somewhat detailed manner. We claim that the bulk viscosity will be generated from the pressure gradient term, when the relaxation between pressure and density fluctuations is correctly handled [3]. It is the delayed response of the density fluctuation [4] to an imposed pressure fluctuation that causes the damping of sound (apart from the purely Stokes damping). This means that apart from the Stokes damping, all other damping should emerge from the two linear equations

$$\frac{\partial}{\partial t}\delta\rho+\rho_0(\vec{\nabla}\cdot\vec{v})=0 \tag{2}$$

and

$$\frac{\partial\vec{v}}{\partial t}=-\frac{\vec{\nabla}\delta P}{\rho_0} \tag{3}$$

where $\delta\rho$ and δP are fluctuations in density and pressure about a constant background. In terms of the density and entropy fluctuations

$$\delta P=\left(\frac{\partial P}{\partial\rho}\right)_S\delta\rho+\left(\frac{\partial P}{\partial S}\right)_\rho\delta S=\kappa_S\delta\rho+\left(\frac{\partial P}{\partial S}\right)_\rho\delta S \tag{4}$$

where $\kappa_S=\left(\frac{\partial P}{\partial\rho}\right)_S$ is the isentropic bulk modulus.

A time derivative of Eq.(3) yields

$$\frac{\partial^2\vec{v}}{\partial t^2}=\vec{\nabla}\left(\kappa_S(\vec{\nabla}\cdot\vec{v})\right)-\vec{\nabla}\left(\frac{\partial P}{\partial S}\right)_\rho\frac{1}{\rho_0}\delta\dot{S} \tag{5}$$

The second term on the right hand side of the above equation is a damping term and can be shown to give rise to the Kirchoff

damping [5], which we will ignore here. The first term on the right side is the sound speed, if κ_S is a constant. This is generally true except when the system is very close to its critical point. Near the critical point, the relaxation of the density fluctuations is very slow [6,7] and the frequency of the sound can resonate [8] with the decay rate of the fluctuations. This gives rise to a frequency dependence in κ_S that now has real and imaginary parts and can be written as

$$\kappa_S = \kappa_S^r + i\omega\kappa_S^{im} \tag{6}$$

where κ_S^r and κ_S^{im} are respectively the real and imaginary parts of κ_S .

With the above discussion in mind, we can now write Eqs. (2) and (3) as

$$\frac{\partial^2\rho}{\partial t^2} = -\rho_0\vec{\nabla}\cdot\dot{\vec{v}} = \vec{\nabla}\cdot\left(\vec{\nabla}\delta P\right) = \kappa_S\nabla^2\delta\rho \tag{7}$$

The term on the right hand side of Eq.(7) leads to damping because of the iω factor in Eq. (6) and defines the bulk viscosity as

$$\eta_B = \rho\kappa_S^{im}(\omega) \tag{8}$$

The dispersion relation that we get from Eq.(7) is

$$\omega^2 = C_S^2k^2 + i\omega\kappa_S^{im}k^2 \tag{9}$$

where k is the wave vector and $C_S^2 = \kappa_S^r(\omega)$. Writing $\omega = C_S(k_1 + ik_2)$, equating real and imaginary parts in Eq. (9), and remembering $k_2 \ll k_1$, we find $\omega^2 = C_S^2k_1^2 + i\omega\kappa_S^{im} = C_S^2(k_1^2 - k_2^2 + 2ik_1k_2) = C_S^2k_1^2 + 2ik_1k_2C_S^2$, which leads to $\frac{k_2}{k_1} = \frac{\omega\kappa_S^{im}}{2C_S^2}$. The attenuation per wavelength is k_2 and hence the attenuation per wavelength is

$$\alpha_\lambda = \frac{k_2}{k_1} = \frac{\omega\kappa_S^{im}}{2C_S^2} \tag{10}$$

This attenuation comes entirely from critical fluctuations.

For very low frequency, i.e., $\omega \ll \Gamma_0\kappa^z$, we expect the attenuation to be linear in the reduced frequency $\frac{\omega}{\Gamma_0\kappa^z}$ and hence

$$\alpha_\lambda = \alpha_0\frac{\omega}{2\Gamma_0\kappa^z} \tag{11}$$

Where α_0 is a constant, $= \frac{1}{\xi}$, and $z = 3 + x_\eta$ is the dynamic scaling exponent. As a first approximation, we will drop x_η, which is very very small. . From equations (11), (10), (8) and (1), we get

$$R_B = \alpha_0 \tag{12}$$

We now note that near the critical point T_c, the isothermal compressibility, κ_T $\left[= -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T\right]$ is very large and so the sound speed can be written as

$$C_S^2 = \frac{1}{\rho\kappa_T} + T\frac{\left(\frac{\partial P}{\partial T}\right)_V^2}{\rho^2C_V} \cong T\frac{\left(\frac{\partial P}{\partial T}\right)_V^2}{\rho^2C_V} \tag{13}$$

where C_V is the constant volume specific heat. This is the only critical quantity in the above expression. Both ρ (one phase region) and $\left(\frac{\partial P}{\partial T}\right)_V$ are noncritical. The response function, which is the specific heat, exhibits a lagging effect near the critical point and hence C_V is the frequency dependent. Consequently, we can write

$$C_V = C_V^r + iC_V^{im} \tag{14}$$

and the use of Eqs. (13) and (14) leads to

$C_S^2 = T\frac{\left(\frac{\partial P}{\partial T}\right)_V^2}{\rho^2}\frac{C_V^r - iC_V^{im}}{\left(C_V^r\right)^2 + \left(C_V^{im}\right)^2}$. Since $C_V^r \gg C_V^{im}$, we can write $C_S^2 = T\frac{\left(\frac{\partial P}{\partial T}\right)_V^2}{\rho^2}\frac{C_V^r - iC_V^{im}}{\left(C_V^r\right)^2}$. Now $C_S^2 = \frac{\omega^2}{k^2} = \frac{\omega^2}{k_1^2\left(1-2i\frac{k_2}{k_1}\right)}$ and comparing with the previous form, we get

$$\alpha_\lambda = \frac{k_2}{k_1} = \frac{1}{2}\frac{C_V^{im}}{C_V^r} \tag{15}$$

We now need to discuss the scaling behaviour of C_V. At zero frequency, the specific heat diverges near the critical point as $\xi^{\alpha/\nu}$, where α is a small exponent (for the gas-liquid critical point $\alpha \approx 0.11$). Thus

$$C_V(\omega = 0, \kappa) = C_0\frac{\kappa^{-\alpha/\nu} - 1}{\frac{\alpha}{\nu}} \tag{16}$$

At finite frequency, the response is limited by the finite frequency as one approaches the critical point. As we lower the value of κ, at a finite ω, there comes a temperature at which $\Gamma_0\,\kappa^z \sim \omega$ and if κ (temperature) is further lowered, the response cannot change any more and we have the dynamic scaling result [9]

$$C_V(\omega, \kappa = 0) = C_0\frac{(-i\omega)^{-\alpha/z\nu} - 1}{\frac{\alpha}{\nu}} \tag{17}$$

The passage from Eq.(16) to Eq.(17) is determined by the scaling function $F\left(\frac{\omega}{\Gamma_0\kappa^z}\right)$, such that $C_V(\kappa, \omega)$ can be written as

$$C_V(\kappa, \omega) = C_0\frac{\kappa^{-\alpha/\nu}F(\Omega) - 1}{\frac{\alpha}{\nu}} \tag{18}$$

where $\Omega = \frac{\omega}{2\Gamma_0\kappa^z}$ and $\Omega \gg 1$, $F(\Omega) \sim (-i\omega)^{-\alpha/z\nu}$ to re-produce Eq.(17). The simplest form of $F(\Omega)$, which is inspired by an exponentiation scheme due to Nicol for the wave number dependent specific heat is

$$C_V(\kappa, \omega) = C_0\frac{\kappa^{-\alpha/\nu}[1 - i\beta\Omega]^{-\alpha/\nu} - 1}{\frac{\alpha}{\nu}} \tag{19}$$

Where β is a number of the order of unity, which can be fixed from the low frequency behaviour in Ω or the high frequency behaviour. In this case, we imagine β determined by the low frequency expansion of the diagrammatic perturbation series for $C_V(\kappa,\omega)$. Expanding Eq.(19) for Ω » 1, we find

$$\alpha_\lambda = \beta\frac{\alpha}{\nu}\frac{\omega}{2\Gamma_0\kappa^z}\frac{\frac{\alpha}{\nu}}{\kappa^{-\alpha/\nu} - 1} \cong \beta\frac{\alpha}{\nu}\frac{\omega}{2\Gamma_0\kappa^z}\frac{1}{ln\frac{1}{\kappa}}, \quad \text{for } \frac{\alpha}{\nu} \ll 1 \tag{20}$$

In a one loop calculation [5], $\beta\left(\frac{\alpha}{\nu}\right) = \frac{4}{3\pi}$. Comparing with Eqs. (1), (11) and (12), we find

$$R_B = \frac{4}{3\pi}\frac{1}{ln\frac{\kappa_0}{\kappa}} \tag{21}$$

This is the principal result that we wanted to show. R_B is not a strict constant, but has a weak dependence on κ (temperature) through the logarithmic factor.

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